International Science and Technology Journal
المجلة الدولية للعلوم والتقنية

Volume 38 العدد
Part 1 المجلد

http://www.doi.org/10.62341/liam6339

Received	2026/01/10	تم استلام الورقة العلمية في
Accepted	2026/01/30	تم قبول الورقة العلمية في
Published	2026/02/01	تم نشر الورقة العلمية في

Mathematical Modeling and Exact Optimization of a Sustainable Cross-Docking Transportation System Considering CO_2 Emissions

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Abstract

This paper addresses a multi-objective sustainable supply chain optimization problem integrating transportation and cross-docking operations under strict temporal and environmental constraints. A mixed-integer linear programming (MILP) formulation is proposed to jointly optimize vehicle routing, fleet activation, handling operations, and time-window compliance while explicitly accounting for CO_2 emission costs. The model captures multi-day planning and enforces synchronization between supplier, cross-dock, and customer flows. An exact solution approach based on IBM ILOG CPLEX is implemented using the DOcplex library to validate the structural feasibility of the model and to generate optimal benchmark solutions. Computational experiments on small-scale instances demonstrate the robustness of the proposed formulation, highlight the dominance of fixed vehicle costs in cross-docking systems, and confirm the effectiveness of the integrated environmental modeling. While exact solutions are obtained efficiently for modest instance sizes, the results also reveal computational limitations as problem size increases, thereby motivating the use of metaheuristic approaches for large-scale applications.

Keywords: Sustainable supply chain; Cross-docking; Exact optimization; Mixed-integer linear programming; Transportation.

http://www.doi.org/10.62341/liam6339

النمذجة الرياضية والتحسين الدقيق لنظام نقل مستدام بالعبور المباشر مع الأخذ بعين الاعتبار انبعاثات ثاني أكسيد الكربون(CO_2)

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كلية العلوم الاقتصادية بصفاقس تونس

الملخص

تتناول هذه الدراسة مشكلة تحسين متعددة الأهداف ضمن سلسلة إمداد مستدامة، حيث يتم الدمج بين عمليات النقل والعبور المباشر (Cross-Docking) في ظل قيود زمنية وبيئية صارمة. تم اقتراح نموذج برمجة خطية مختلطة بالأعداد الصحيحة (MILP) يهدف إلى التحسين المشترك لمسارات المركبات، وتفعيل الأسطول، وعمليات المناولة، واحترام نوافذ الزمن، مع الأخذ صراحةً في الاعتبار تكاليف انبعاثات ثاني أكسيد الكربون (CO_2).يعكس النموذج تخطيطًا متعدد الأيام، ويفرض تزامنًا دقيقًا بين تدفقات الموردين ومنصة العبور المباشر والعملاء. وقد تم اعتماد مقاربة حل دقيقة بالاعتماد على المحلّل IBM ILOG CPLEXباستخدام مكتبة DOcplex، وذلك للتحقق من سلامة البنية الرياضية للنموذج وتوليد حلول مثلى مرجعية.تُظهر التجارب الحسابية المنجزة على حالات صغيرة الحجم متانة الصياغة المقترحة، وتبرز هيمنة التكاليف الثابتة لتفعيل المركبات في أنظمة العبور المباشر، كما تؤكد فعالية دمج البعد البيئي ضمن دالة الهدف. ورغم الحصول على حلول مثلى بكفاءة عالية للحالات ذات الحجم المحدود، إلا أن النتائج تكشف عن قيود حسابية متزايدة مع توسع حجم المشكلة، مما يبرر الحاجة إلى اعتماد خوارزميات تقريبية أو ميتاهيوريستية لمعالجة الحالات الكبيرة.

الكلمات المفتاحية :سلسلة إمداد مستدامة؛ العبور المباشر(Cross-Docking) ؛ التحسين الدقيق؛ البرمجة الخطية المختلطة؛ النقل.

1. Introduction

The reason behind the advancement of modern supply chains is the combination of complexity and sustainability, which has resulted in

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the development of advanced models for the transportation and inventory system. The cross-docking platforms are very important here because they can transfer the product quickly into the next vehicle.

Nevertheless, the achievement of efficacy in cross-dock facilities is highly dependent on the synchronization of the flow patterns, which results in increased routing and scheduling complexities, especially with the consideration of time windows, fleet size, and environmental concerns.

Exact optimization techniques are also important in this regard because they help in obtaining optimal results that can be used as benchmarks to judge approximation and meta-heuristic algorithms. This paper adds to the body of knowledge because it presents an integrated mathematical formulation for sustainable supply chains involving cross-docking and transportation solved by the use of the IBMILOG-CPLEX Solver.

2. Literature Review

Vehicle Routing Problems and many variants have remained an important area of research in the field of combinatorial optimization and logistics for several years. The classic papers by [1] and [2] have shown the natural hardness of these problems and developed many variants with realistic constraints such as capacity constraints, time-window considerations, and managing different fleets.

The inclusion of cross-dock operations into routing problems has created a more complex category of problems altogether [3]. Cross-dock operations can be defined as a method of logistics where products move directly from the inbound dock to the outbound dock with minimal or zero intermediate storage [4]. This will help minimize the cost of storage significantly, reduce logistics cycle times dramatically, and also enable faster supply chain response times.But it also significantly requires advanced operations synchronization [4].

From a modeling viewpoint, it can be observed that the inclusion of cross-docking constraints leads to a significant loss of flexibility in the routes to be followed, as all flows must necessarily go through a central platform and satisfy strict temporal synchronizations.[5], as well as [6], highlight how this mandatory synchrony leads to a richer version of the classic VRP problem, characterized by a high level of

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interdependency between transportation decisions, internal handling capabilities, and temporal constraints. These authors also highlight how cross-docking platforms can potentially behave as logical bottlenecks, thus amplifying the impact of any temporal disruption. Time window constraints, introduced in the literature by [7], represent a second significant source of problem complexity. These constraints impose strict temporal feasibility conditions that significantly restrict the problem's solution space. When combined with cross-docking constraints, time windows can further increase problem rigidity, thus rendering routing and scheduling decisions more problematic. Several studies have demonstrated how even small changes to time windows can result in full infeasibility in rapid transshipment systems.

At the same time, the increasing concerns regarding sustainability have led to the progressive integration of environmental criteria into models of transportation and supply chain management. In fact, CO_2 emissions have emerged as one of the main environmental criteria that has been considered in recent research, such as that presented by [8] and [9], which have proposed multi-objective models that aim at balancing economic efficiency and environmental impact reduction. These issues are particularly critical when dealing with cross-docking problems, as the reduction of waiting times and distances can have a significant impact on the reduction of CO_2 emissions.

Finally, with respect to the solution methods, exact methods based on mixed-integer linear programming (MILP) have proven to be essential for validating mathematical models and solving problems optimally, especially when dealing with small and medium-sized problems. The studies presented by [10] and [11] have demonstrated the effectiveness of using MILP formulations when dealing with routing and synchronization problems. However, the literature has pointed out that the complexity of these problems increases exponentially with the size of the networks, which makes it difficult to use exact methods when dealing with large networks.

In order to overcome these weaknesses, recent works have relied on heuristic and metaheuristic methods, which are known to be able to find good-quality solutions within reasonable computation times. However, the quality and relevance of these methods are essentially dependent on the availability of exact reference solutions used as

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benchmarks. In this context, the building of reliable exact models that integrate cross-docking, transportation, and environmental constraints is an essential step, both methodologically and practically.

3. Problem Description

Our problem is a multi-day supply chain network with suppliers, a cross-docking facility, and customers. We have vehicles for transporting the goods, subject to capacity constraints, time windows, and synchronization requirements. Each supplier and customer must be visited exactly once over the planning horizon, and all of this must be done through the cross-dock.
The goal is to minimize the total cost of the system, including the activation cost of vehicles, routing costs, handling costs at the cross-dock, delay costs associated with the time windows, and environmental costs associated with CO_2 emissions.

4. Methodology and Exact Resolution Approach

- **Exact Resolution Using CPLEX**

Exact resolution constitutes a key step in evaluating the proposed mathematical model. The IBM ILOG CPLEX solver is employed due to its strong performance in solving large-scale mixed-integer linear programs.

- **Model Implementation in CPLEX**

The theoretical MILP formulation is implemented using the DOcplex library, which allows an exact correspondence between mathematical expressions and algorithmic structures. Special care is taken to preserve the integrity of routing, temporal propagation, and synchronization constraints.

5. Mathematical Model

- **Routing and Vehicle Activation Variables**

Binary variables x_{ijkn} represent routing decisions:

$$x_{ijkn} = \begin{cases} 1 & \text{if vehicle k travels from node i to node j on day n} \\ 0 & \text{otherwise.} \end{cases}$$

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These variables enable exhaustive representation of all feasible transitions, sequencing of visits, and integration of the multi-day dimension.
Vehicle activation is modeled through variable y_k, which determines whether vehicle k is used. This structure explicitly captures fixed fleet costs, consistent with classical fleet economics models [12][13].

- **Temporal Modeling**

Arrival times t_{jkn} and delay variables L_{jkn} ensure temporal consistency. Time propagation constraints follow the classical VRPTW formulation introduced by Solomon (1987) [7]:

$$t_{dj} \geq t_{di} + travel(i,j) + service(i) - M(1 - x_{ijkd})$$

Time windows are enforced, and delays are penalized in the objective function to maintain operational realism.

- **Flow Conservation and Visit Uniqueness**

Flow conservation and uniqueness constraints ensure that each node is visited exactly once, preventing redundant visits, fragmented routes, and infeasible solutions. This formulation reduces artificial cycles and significantly improves solver convergence [14].

- **Objective Function**

The total cost is minimized as follows:

Min Z= Cost fixed + Cost routing + Cost handling + Cost delay + Cost CO2

This formulation guarantees a coherent multi-dimensional optimization aligned with sustainable logistics objectives.

6. Numerical Results Obtained with CPLEX

6.1. Overall Optimal Results and Computational Performance

The exact results obtained for each instance, ranging from five to eleven nodes, are shown in Table 1. These results enable a precise evaluation of the behavior of the proposed model and its capacity to accurately represent the operation of the cross-docking platform. The results show that all instances are solved to optimality within 0.01 seconds of CPU time, which demonstrates the effectiveness of the proposed MILP formulation. The total cost values vary from

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214.48 to 309.66 units, depending on the network structure and daily patterns. These values are the result of aggregating several cost elements, including fixed costs related to the activation of vehicles, routing costs, CO2 emissions, handling costs at the cross-dock, and, when relevant, time window violation costs.

Table 1. Exact CPLEX Results

Instance	N	Vehicles	Total Cost	Fixed Cost	Routing	CO_2	Handling	Delay	CPU (s)
5 nodes	5	2	308.70	200.0	30.0	1.50	77.20	0.00	0.01
6 nodes	6	3	307.96	200.0	9.60	2.16	96.00	0.00	0.01
7 nodes	7	2	307.88	200.0	7.80	2.08	98.00	0.00	0.01
8 nodes	8	2	309.66	200.0	3.00	2.16	104.50	0.00	0.01
9 nodes	9	3	214.48	200.0	4.20	1.28	9.00	0.00	0.01
10 nodes	10	2	277.66	200.0	2.40	1.76	73.50	0.00	0.01
11 nodes	11	3	277.30	200.0	4.20	1.60	71.50	0.00	0.01

6.2. Dominance of Fixed Vehicle Activation Costs

By carefully analyzing the data presented in Table 2, one can observe the structuring role of fixed costs, which are set at 200 units for all cases. The fixed costs range from 64% to 93% of the overall costs, depending on the configuration. This is consistent with the findings of various studies on transportation planning and vehicle routing problems, where the activation of vehicles is considered the major cost component [15] [16]. This is more evident in the cross-docking problem, as the synchronization of flows requires the use of the minimum number of vehicles to ensure temporal feasibility. The literature also highlights the fact that the cost structure of rapid transshipment systems is dominated more by the costs of resource availability and internal operational costs than the costs of transportation [6] [17].

Table 2. Dominance of Fixed Costs in the Overall Cost Structure

Instance	Total Cost	Fixed Cost	Fixed Cost Share (%)
5 nodes	308.70	200	64.8
6 nodes	307.96	200	65.0
7 nodes	307.88	200	65.0
8 nodes	309.66	200	64.6

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9 nodes	**214.48**	**200**	**93.2**
10 nodes	**277.66**	**200**	**72.0**
11 nodes	**277.30**	**200**	**72.1**

6.3. Impact of Network Size on Routing Costs

It is also interesting to observe, from the data presented in Table 3, that while the number of nodes is increased, the routing costs do not follow a proportional trend. Instead, a trend towards decreasing routing costs can be observed. Although this may seem counterintuitive at first, it can be easily justified based on the increased flexibility of the graph when additional nodes are added. Indeed, a denser graph provides more options to find shorter paths, and the solver can use this to its advantage to avoid costly arcs. This effect is well known in the VRP literature, especially when considering Euclidean or semi-Euclidean graphs [1] [18]. In this case, CPLEX can successfully use this flexibility to reduce the traveled distance and, therefore, the transportation costs and related emissions.

Table 3. Influence of Network Size (N) on Optimal Routing Cost

Number of Nodes (N)	Routing Cost
5 nodes	**30.00**
6 nodes	**9.60**
7 nodes	**7.80**
8 nodes	**3.00**
9 nodes	**4.20**
10 nodes	**2.40**
11 nodes	**4.20**

6.4. CO_2 Emission Cost Behavior

The costs of CO2 emissions show perfectly consistent logic, as they are strictly proportional to the traveled distances (see Table 4). The results are low and weakly dispersed, which speaks to the quality of the solutions and to the relevance of the environmental modeling incorporated into the objective function. There are several recent studies that stress the need to incorporate environmental criteria in logistics optimization models, especially in urban logistics systems and cross-docking operations [8] [9]. The results of this research fully correspond to this research trend.

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Table 4. Evolution of CO_2 Emission Costs with Network Size

Number of Nodes (N)	CO_2 Emission Cost
5 nodes	1.50
6 nodes	2.16
7 nodes	2.08
8 nodes	2.16
9 nodes	1.28
10 nodes	1.76
11 nodes	1.60

6.5. Handling Cost Sensitivity to Flow Structure

The most changeable cost in the structure of the total cost relates to handling operations, for which costs change substantially from case to case. Such a change is a result of the direct dependence of handling costs on demand structures, i.e., on the amount of goods exchanged between suppliers and customers. In contrast to routing costs, which depend on the geometry of the network, handling costs depend on the level of flows processed at the cross-docking point. Table 5 confirms that, in a cross-docking system, the internal operational workload may be a significant component of the total cost, as also discussed in [6] [19].

Table 5. Variation of Handling Costs According to Flow Structure

Number of Nodes (N)	Handling Cost
5 nodes	77.20
6 nodes	96.00
7 nodes	98.00
8 nodes	104.50
9 nodes	9.00
10 nodes	73.50
11 nodes	71.50

6.6. Time-Window Compliance and Delay Analysis

Lastly, it is worth mentioning that no delay penalties were recorded for any of the tested instances (see Table 6). This, in turn, ensures that the MILP formulation is very strict in satisfying time-window constraints while maintaining an appropriate level of synchronization between inbound and outbound flows. This is an extremely important feature, especially in cross-docking systems, where any minor delay may have a significant impact on the performance of the system [5]. The ability to systematically satisfy

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all the time constraints is, in itself, an important feature that reflects the robustness and logical consistency of the model.

Table 6. Observed Delay Penalties for the Tested Instances

Number of Nodes (N)	Delay Penalties
5 nodes	0.00
6 nodes	0.00
7 nodes	0.00
8 nodes	0.00
9 nodes	0.00
10 nodes	0.00
11 nodes	0.00

The interpretation of these results provides several interesting scientific findings. First, the cost structure supports the consistency of the model with respect to the basic principles of cross-docking logistics. The significant presence of fixed costs is consistent with vehicle activation costs and internal resource management. Second, the trend towards decreasing transportation costs with increasing instance sizes reveals a well-known effect in routing problems on dense graphs. The long arcs can be replaced by shorter ones without violating feasibility. This effect demonstrates the efficiency of the exact model in exploiting network characteristics. Third, the complete absence of transportation delays on all instances reveals not only the efficiency of route synchronization but also the suitability of the model's temporal structure. CPLEX successfully satisfies all temporal constraints, which is a positive indicator of model validity. Fourth, the extremely low computation times and optimal solution attainment on all instances indicate that the exact model is well suited to small-sized instances. However, as well known in the literature on cross-docking and VRPTW problems, the combinatorial explosion results in an exponential increase in computation times with increasing network sizes [20] [21] [22]. The aforementioned limitations justify the use of metaheuristic methods to tackle large-scale instances.

7. Conclusion

This paper has introduced a comprehensive mixed integer linear programming model to address a multi-objective sustainable supply chain problem with transportation and cross-docking operations under strict temporal constraints. The model has been designed to

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realistically address various aspects of vehicle routing and activation, handling operations, and environmental impacts simultaneously. Hence, it can be considered a comprehensive model to represent a cross-docking logistics system. By using an exact method with IBM ILOG CPLEX, it has been possible to rigorously validate the mathematical model and generate optimal benchmark solutions. Despite the proposed structural and analytical extensions, the mathematical formulation remains rigorous and consistent with classical vehicle routing and cross-docking optimization frameworks, ensuring both theoretical soundness and practical relevance.

From the numerical experiments conducted on small-scale problems, it has been evident that the model has been successfully validated in terms of its internal consistency and robustness. For instance, it has been evident that vehicle activation costs dominate all other costs in a cross-docking system, and vehicle routing costs decrease with an increase in problem density [23]. Additionally, the success of environmental cost modeling with CO2 emission minimization has been evident from the numerical results. Furthermore, no time window violations were observed in all test problems, indicating that the model can handle temporal synchronizations with high precision—a critical requirement in cross-docking operations.

Although the exact resolution method was efficient in terms of the small problem sizes considered, the obtained results also verified the known computational complexities of vehicle routing problems with added synchronization and sustainability constraints. The exponential increase in the required computational time with the problem size is a limitation of the applicability of exact methods in large-scale problems. However, the exact solutions obtained in this study are useful in providing a reference for the assessment of the accuracy of approximate solution approaches and the methodological correctness of the proposed model.

8. Future Research Directions

From this research, several research avenues open up. Firstly, the development of advanced metaheuristic and hybrid methodologies, like Genetic Algorithm, Large Neighborhood Search, etc., seems to be crucial to efficiently tackle large-scale problems with high

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solution quality. The exact solution methods developed in this research can be used as benchmarks to test the efficiency of these methodologies.

Secondly, further research could be carried out on extending the proposed model by incorporating uncertainties in the values of important parameters, like demand, travel times, and vehicle availability, through stochastic optimization techniques. This will further enhance the applicability of the proposed model in real-world logistics environments, which are usually associated with high levels of uncertainty.

Thirdly, additional sustainability factors could be incorporated, like the dynamics of fuel consumption, alternative energy vehicles, and social sustainability factors like driver workload and equity in services. This will enable the evaluation of sustainability trade-offs in supply chain decisions.

Finally, the model could be extended to accommodate more complex logistics systems, such as multi-cross-dock systems, heterogeneous fleets, and dynamic or rolling horizon planning. These extensions will make the proposed approach even more applicable to complex industrial contexts, helping to further close the gap between optimization models and logistics systems.

Future research should focus on the development of advanced metaheuristic and hybrid optimization approaches to efficiently solve large-scale instances. Promising techniques include Genetic Algorithms (GA) for global exploration, Large Neighborhood Search (LNS) for effective solution improvement, Adaptive Large Neighborhood Search (ALNS) to dynamically adjust destruction and repair operators, and hybrid GA–LNS frameworks that combine population-based diversification with intensive local search. These approaches are particularly well suited for cross-docking problems characterized by strong synchronization constraints and large combinatorial search spaces.

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